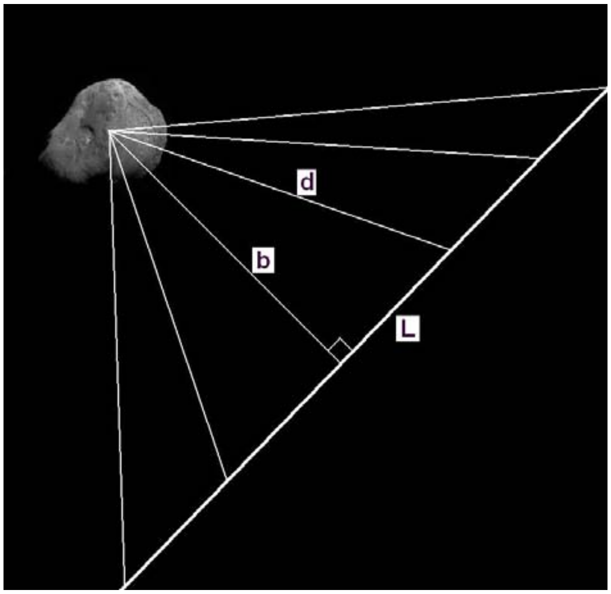
Deep Impact - Comet Flyby



On July 4, 2005, the Deep Impact spacecraft flew by the comet Tempel-1 along a path, L, shown in the figure to the left, at a speed of 10 km/sec. Its closest distance to the comet was b = 500 kilometers at a time, t=0. The distance traveled along the path is given by l = Vt.

The diameter of the comet is L = 8 kilometers, and the distance to the comet in kilometers is d(t), so the angular diameter of the comet in arcminutes is given by

$$\Theta(t) = \frac{3438L}{d(t)}$$

Problem 1 - What is the formula for the distance to the comet from the spacecraft defined as *d(t)*?

Problem 2 - What is the formula for the angular diameter of the comet as seen from the spacecraft at any time, t, along the trajectory, defined by the variables V, b and L? Simplify the formula by defining two constants B = 3438L/b and c = V^2/b^2.

Problem 3 - What is the exact numerical formula for the rate-of-change in time of the angular size of the Tempel-1 as viewed by the spacecraft as it flys-by?

Problem 4 - What was the diameter of Tempel-1 at the closest approach, and how fast will the angular size be decreasing when Tempel-1 reaches one-half its maximum angular size?

Problem 1 - What is the formula for the distance to the comet from the spacecraft defined as d(t)? Answer: Use the Pythagorean Theorem and the diagram to determine that

$$d(t) = \sqrt{b^2 + V^2 t^2}$$

Problem 2 - What is the formula for the angular diameter of the comet as seen from the spacecraft at any time, t, along the trajectory? Simplify the formula for Θ(t) by defining two constants B = 3438L/b and $c = V^2/b^2$. Answer:

$$\Theta(t) = \frac{B}{\sqrt{1+ct^2}}$$

Problem 3 - What is the exact numerical formula for the rate-of-change in time of the angular size of the Tempel-1 as viewed by the spacecraft as it flys-by? Answer; Evaluate B and c for the specific values of L, b and V for the Tempel-1 flyby. Remember to use consistent units (all units in meters, and meters/sec). B = 3438 x 8,000 meters/(500,000 meters) so B = 55 arcminutes, and c = $(10{,}000 \text{ meters/sec})^2$/(500,000 meters) so $c = 0.04 \text{ sec}^{-2}$. Then the formula becomes,

$$\Theta(t) = \frac{55}{\sqrt{1+0.04t^2}}$$

where θ(t) will be the comet diameter in arcminutes. The rate-of-change of Θ(t) with respect to time is just its first-derivative, which we then evaluate for B=55 and c=0.04. The formula will provide answers in units of arcminutes/sec:

$$\frac{d\Theta(t)}{dt} = -\frac{1}{2}B\left(1+ct^2\right)^{-3/2}(2ct)$$

or

$$\frac{d\Theta(t)}{dt} = -\frac{2.2t}{\left(1+0.04t^2\right)^{3/2}}$$

Problem 4 - What was the diameter of Tempel-1 at the closest approach, and how fast will the angular size be decreasing when Tempel-1 reaches one-half its maximum angular size?

Answer: For t=0 we get $\theta(0) = 55$ arcminutes. To decrease by a factor of two its final diameter has to be 27.5 arcminutes, so θ(t)=27.5, and solve for t to get t= 8.7 seconds.

From the derivative formula, and evaluating it at t=8.7 seconds, we find that dθ(8.7 sec)/dt = 2.4 arcminutes/sec.