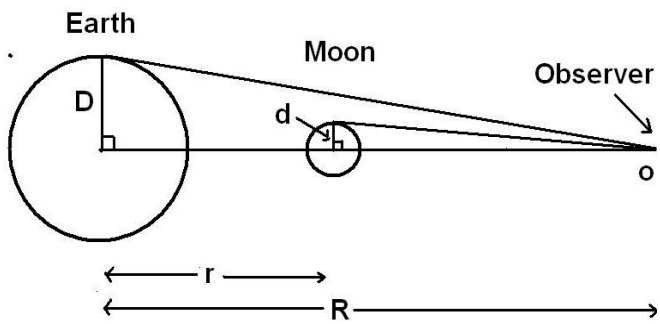
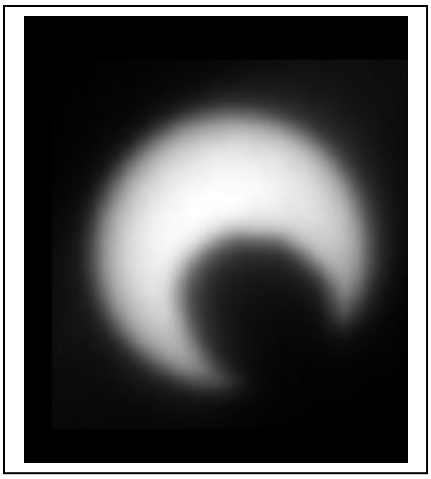
Earth and Moon Angular Sizes



In space, your perspective can change in complicated ways that sometimes go against Common Sense unless you 'do the math'. This happens very commonly when we are looking at one object pass across the face of another. Even though the Moon is 1/4 the diameter of Earth, the simple ratio of their apparent diameters will depend on how far from them YOU are when you see them.

An important 'skinny triangle' relationship for triangles states that, if the angle is less than 1 degree (< 0.017 radians), the angle measure in radians equals very nearly the sine of the angle, which is just the ratio of the opposite side to the hypotenuse: $\theta = D/R$ where D is the radius of the object in kilometers, R, is the distance to the object in kilometers, and θ is the angular radius of the object in radians. For instance, the Moon is located r = 384,000 km from Earth and it has a radius of d = 1,738 km, so its angular radius is 1,738/384,000 = 0.0045 radians. Since 1 radian = 57.3 degrees, the angular radius of the Moon is 0.0045 x 57.3 = 0.26 degrees, so its diameter is 0.52 degrees as viewed from Earth.



Problem 1 - In the figure above, assume that the diameter of the Moon is less than 1 degree when spotted by the spacecraft located at 'O'. What is the angular diameter of: A) the Moon, θ_m, in terms of d and R? B) The Earth, θ_e, in terms of D and R? and C) What is the ratio of the angular diameter of the Moon to the Earth in terms of d, D, r and R?

Problem 2 - A spacecraft is headed directly away from the Moon along the line connecting the center of Earth and the Moon. At what distance will the angular diameter of the Moon equal the angular diameter of Earth?

Problem 3 - The figure to the top left shows the martian satellite Phobos passing across the disk of the sun as viewed from the surface of Mars by the Rover Opportunity. If the ratio of the diameters is 1/2, and if r = 228 million km, d = 10 km, and D=696,000 km, about how far from Phobos was Opportunity at the time the photo was taken?

Problem 4 - The Deep Impact spacecraft observed the Moon pass across the disk of Earth as shown in the photo to the bottom left. The ratio of the disk diameters is 1/3.9, and if r = 384,000 km, d=1,786 km and D = 6,378 km, about how far from Earth, R, was the spacecraft?

Problem 5 - As the distance, R, becomes very large, in the limit, what does the angular ratio of the disk approach in the equation defined in Problem 1?

Problem 1 - What is the angular diameter of:
A) the Moon, θ_m, in terms of d and R? Answer:

$$\theta m = \frac{2\pi d}{R-r}$$

B) The Earth, θ_e, in terms of D and R? Answer:

$$\theta e = \frac{2\pi D}{R}$$

C) What is the ratio of the angular diameter of the Moon to the Earth in terms of d, D, r and R? Answer:

$$\frac{\theta m}{\theta e} = \frac{R}{R-r}\frac{d}{D}$$

Problem 2 - At what distance will the angular diameter of the Moon equal the angular diameter of Earth? Answer:

$$1 = \frac{R}{R-384,000}\frac{(1,786)}{(6,378)}$$

so 3/4 R = 384,000 km, and so R = 512,000 km.

Problem 3 - If the ratio of the diameters is 1/2, and if r = 228 million km, d = 10 km, and D=696,000 km, about how far from Phobos was Opportunity at the time the photo was taken? Answer:

$$\frac{1}{2} = \frac{R}{R-228million}\frac{(10km)}{(696,000km)}$$

so 34799 R = 228 million km, and so R = 6,552 km.

Problem 4 - The ratio of the disk diameters is 1/3.9, and if r = 384,000 km, d=1,786 km and D = 6,378 km, about how far from Earth, R, was the spacecraft? Answer:

$$\frac{1}{3.9} = \frac{R}{R-384,000km}\frac{(1,786km)}{(6,378km)}$$

so 0.02 R = 384,000 km , and so R = 19.2 million km.
Note: the actual distance was about 30 million km for the photo shown in this problem.

Problem 5 - As the distance, R, becomes very large, in the limit, what does the angular ratio of the disk approach in the equation defined in Problem 1? Answer: As R becomes much, much larger than r (e.g the limit of r approaches infinity), then the equation approaches

$$\frac{\theta m}{\theta e} = \frac{d}{R}\frac{R}{D}$$

and since the 'R' terms cancel, we get the angular ratio approaching the physical ratio d/D of the diameters of the two bodies. In other words, although the apparent angular sizes change rapidly when you are very close to the bodies and the value of R is comparable to 'r', at very great distances, the angular ratio approaches a constant value d/D. This has many practical consequences in the search for planets around other stars as they' transit' their stars.